Transverse dynamics of a collisionless plasma column in a homogeneous magnetic field

D. S. Dorozhkina and V. E. Semenov
Institute of Applied Physics, RAS, 46 Ulyanov Street, 603600 Nizhny Novgorod, Russia
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Results of a general study of a set of kinetic equations describing the dynamics of a two-component quasineutral collisionless plasma column in a homogeneous, nonstationary magnetic field are presented. The dynamics of the column is found to be completely determined by initial values of total kinetic energy of a plasma, total angular momentum of each plasma component, and total mass of a plasma. In the case of a stationary magnetic field the column cross section is shown to oscillate harmonically at the low-hybrid frequency. The peculiarities of the plasma dynamics found are confirmed by self-similar solutions of a set of two Vlasov kinetic equations.

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INTRODUCTION

Plasma expansion into a vacuum has been analyzed by a great number of physicists in the three last decades [1–22]. Following the pioneering work by Gurevich [1], most of the theoretical studies of the problem have been based on the model of semi-infinite collisionless plasma expanding in the absence of a magnetic field. Within this model the acceleration of ions leads to formation of an ion distribution function that is excessively enriched with energetic particles in comparison with the electron distribution function. Such a feature results from an unlimited energy resource in initial plasma and can be expected in experiments when the plasma is coupled with a plasma source that provides energy to keep electrons from cooling down during expansion. A different situation occurs if one deals with a plasma flame generated by an ultrashort laser pulse. In the latter case, the main part of the process of plasma expansion takes place after the laser is turned off. Therefore, the expansion is accompanied by considerable cooling of plasma electrons. Until recently theoretical studies of the problem have only been carried out within a hydrodynamic approach [16]. Focus on the kinetic equations was made only in the 1990s. First, numerical simulation [17,18] demonstrated a possibility of a self-similar regime of plasma expansion with electrons cooling in time. Then, a series of self-similar solutions was found analytically using different approximations [19–21]. Finally, it was shown [22] that an analytical solution of a set of two Vlasov kinetic equations can be obtained in the three-dimensional case under the quasineutral assumption. The solution has been obtained for arbitrary relationships between the masses and initial thermal energies of plasma particles of different sorts. According to the found solution, an initially confined plasma bunch expands infinitely in the absence of external magnetic field. Both the electrons and the ions cool down in adiabatic manner and their thermal energy transfers to the kinetic energy of plasma fluid motion with the same fluid velocity of electrons and ions.

A significant modification of the plasma bunch dynamics should be expected when an external magnetic field is applied to the system. Such a conclusion has been confirmed by a number of laboratory experiments with laser-produced plasma [23,24]. This case, however, needs further theoretical investigation. With the exception of a few papers [see, for instance, [25] where the magnetohydrodynamics (MHD) model was developed to describe semi-infinite plasma expansion transverse to magnetic field], the main attention of researchers was paid to analysis of plasma diffusion transverse to the magnetic field, equilibrium configurations of a plasma in an external magnetic field and their instabilities. The goal of this research is to obtain a solution to the plasma expansion problem in a plane transverse to the homogeneous external magnetic field. The method of moments of distribution functions is used to solve the problem. This fruitful method was developed in [26] to study a multicomponent plasma expansion, an expansion of a plasma bunch with electric current, and also dynamics of a two-component plasma in external, weakly inhomogeneous potential fields. As will be demonstrated below, the method of moments makes it possible to determine the temporal behavior of plasma column parameters in a given magnetic field. These results were used to obtain analytical solutions of two Vlasov kinetic equations that describe the dynamics of an axially symmetric quasineutral plasma column in a spatially homogeneous magnetic field directed along the axis of symmetry. Specifically, the found solution can explain the plasma oscillations observed in recent experiments [27].

I. BASIC MODEL

The exactly solvable physical model implies that there is a column of collisionless plasma with two sorts of particles (for example, electrons and ions). The column is located in a given homogeneous nonstationary magnetic field directed along the axis of the column z (Fig. 1),

$$\mathbf{B}=B(t)\mathbf{e}_z, \tag{1}$$

where $\mathbf{e}_r, \mathbf{e}_\psi, \mathbf{e}_z$ are unit vectors of the cylindrical system of coordinates (r,ψ,z). The plasma column is considered to be axially symmetric and homogeneous along the z axis. The vector potential $\mathbf{A}$ of the magnetic field and the potential of charge-separation electric field φ, respectively, depend only on one spatial variable r, which is the distance from the axis of symmetry of the column

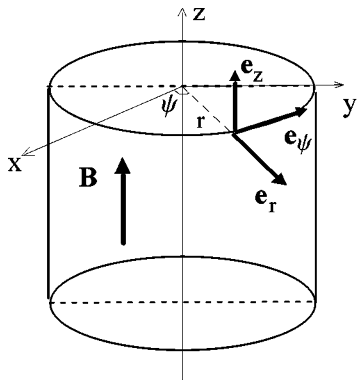

FIG. 1. General configuration of the considered system.

$$\mathbf{B}=[\nabla_{\mathbf{r}}\times\mathbf{A}],\quad \mathbf{A}(\mathbf{r},t)=\frac{1}{2}B(t)r\mathbf{e}_\psi,\quad \varphi=\varphi(r,t),$$
$$\mathbf{r}\equiv(x,y),\quad r\equiv\sqrt{x^2+y^2},\quad \nabla_{\mathbf{r}}\equiv\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y}\right). \tag{2}$$

Particles of each sort are described by distribution functions f_α that do not depend on z. Therefore, they satisfy the following Vlasov kinetic equations:

$$\frac{\partial f_\alpha}{\partial t}+(\mathbf{v}\cdot\nabla_{\mathbf{r}})f_\alpha+\frac{Z_\alpha e}{m_\alpha}(\mathbf{E}\cdot\nabla_{\mathbf{v}})f_\alpha+\frac{Z_\alpha e}{m_\alpha c}([\mathbf{v}\times\mathbf{B}]\cdot\nabla_{\mathbf{v}})f_\alpha=0, \tag{3}$$

$$\mathbf{E}\equiv-\nabla_{\mathbf{r}}\varphi^*\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t},$$

$$\mathbf{v}\equiv(v_x,v_y),\quad \nabla_{\mathbf{v}}\equiv\left(\frac{\partial}{\partial v_x},\frac{\partial}{\partial v_y}\right),$$

where $Z_\alpha e$ and m_α are the charge and mass of particles of sort α, respectively, and c is the velocity of light. The potential of charge-separation electric field φ is found in the course of the solution to the quasineutral approximation that is a commonly accepted approach to analyzing dynamics of a sufficiently dense plasma. It implies that

$$\sum_\alpha Z_\alpha n_\alpha(\mathbf{r},t)=0,\quad n_\alpha(\mathbf{r},t)\equiv\int f_\alpha(\mathbf{v},\mathbf{r},t)d\mathbf{v}, \tag{4}$$

where $n_\alpha(\mathbf{r},t)$ is the density of particles of sort α.

On the one hand, the quasineutral approximation imposes a definite restriction on possible initial distribution functions of particles of different sorts. They should be chosen so as to prevent excitation of Langmuir oscillations in a plasma. On the other hand, the assumption (4) completes the set of basic equations (1)–(4) and makes it possible to obtain an exact solution of the latter.

II. GENERAL ANALYSIS

A fruitful approach to analysis of Eqs. (3) and (4) is based on the method of moments of the distribution functions developed in [26] to investigate the plasma dynamics in free space and in external weakly inhomogeneous potential fields. Specifically, following this method, the spatial structure of a plasma column can be described by the second-order spatial moments:

$$\langle r_k r_j\rangle_\alpha\equiv\frac{1}{N_\alpha}\int\int r_k r_j f_\alpha(\mathbf{v},\mathbf{r},t)d\mathbf{v}d\mathbf{r}, \tag{5}$$

$$N_\alpha\equiv\int\int f_\alpha(\mathbf{v},\mathbf{r},t)d\mathbf{v}d\mathbf{r},$$

where N_α is the total number of particles of sort α per unit length of the plasma column, and r_k, r_j denote the x and y components of the transverse radius vector $\mathbf{r}$. In the case of interest (two-component symmetrical plasma column) the tensors $\langle r_k r_j\rangle_\alpha$ are equal for different sorts of particles due to the quasineutral approximation (4) and can be expressed via one scalar function of time

$$\langle r_k r_j\rangle_\alpha=\frac{1}{2}l^2(t)\delta_{kj}, \tag{6}$$

$$l^2(t)\equiv\langle r^2\rangle\equiv\langle x^2+y^2\rangle_\alpha, \tag{7}$$

where δ_{kj} is the Kroneker symbol, i.e.,

$$\delta_{kj}=1\quad\text{if}\quad k=j,\qquad \delta_{kj}=0\quad\text{if}\quad k\neq j.$$

According to Eq. (7) the function $l(t)$ can be treated as a characteristic spatial scale of the column cross section. The evolution of the latter can be found by integrating the kinetic equations (3) with multiplier r^2:

$$\frac{d}{dt}l^2=2\langle rv_r\rangle,\quad \langle rv_r\rangle\equiv\langle xv_x+yv_y\rangle_\alpha, \tag{8}$$

where the combined coordinate-velocity moment $\langle rv_r\rangle$ is defined by analogy with Eq. (5). Note that according to Eq. (8) it should be the same for different species of the plasma. The subsequent integration of Eqs. (3) results in the following set of equations for the second-order moments:

$$\frac{d}{dt}\langle rv_r\rangle=\sum_\alpha\mu_\alpha\langle v^2\rangle_\alpha+\sum_\alpha\mu_\alpha\omega_\alpha\langle rv_\psi\rangle_\alpha, \tag{9}$$

$$\frac{d}{dt}\sum_\alpha\mu_\alpha\langle v^2\rangle_\alpha=-\sum_\alpha\mu_\alpha\frac{d\omega_\alpha}{dt}\langle rv_\psi\rangle_\alpha, \tag{10}$$

$$\frac{d}{dt}\langle rv_\psi\rangle_\alpha=-\omega_\alpha\langle rv_r\rangle-\frac{1}{2}\frac{d\omega_\alpha}{dt}\langle r^2\rangle, \tag{11}$$

$$\mu_\alpha\equiv\frac{m_\alpha N_\alpha}{M},\quad M\equiv\sum_\alpha m_\alpha N_\alpha,\quad \omega_\alpha\equiv\frac{Z_\alpha e}{m_\alpha c}B(t),$$

where M is the total mass per unit length of the plasma column and ω_α is the cyclotron frequency of particles of sort α. Equations (9)–(11) include moments $\langle v^2\rangle_\alpha$ and $\langle rv_\psi\rangle_\alpha$, which are defined, by analogy with $\langle r^2\rangle, \langle rv_r\rangle$, as

$$\langle v^2\rangle_\alpha\equiv\langle v_x^2+v_y^2\rangle_\alpha,\quad \langle rv_\psi\rangle_\alpha\equiv\langle xv_y-yv_x\rangle_\alpha. \tag{12}$$

Note that these new moments may not coincide for different species in contrast to the previous ones $\langle r^2\rangle,\langle rv_r\rangle$.

It is important that, unlike the well-known hydrodynamic description, the full moments of distribution functions have been used for the analysis. This means integration not only over velocity **v** but also over coordinates **r**. The advantage of such an approach is a finite set of equations for the moments (8)–(11).

Equations (8)–(11) have transparent integrals corresponding to conservation of the z component of the total canonical angular momentum of particles of each sort α:

$$J_\alpha \equiv m_\alpha N_\alpha \left[\langle r v_\psi \rangle_\alpha + \frac{\omega_\alpha}{2}\langle r^2\rangle \right] = \text{const.} \tag{13}$$

Using these integrals it is possible to obtain from the system (8)–(11) one third-order differential equation for the plasma spatial scale l,

$$\frac{d^3}{dt^3} l^2 + \Omega \frac{d}{dt}(\Omega l^2) = 0, \quad \Omega^2 \equiv \sum_\alpha \mu_\alpha \omega_\alpha^2, \tag{14}$$

where the frequency Ω is equal to the low-hybrid frequency for a sufficiently dense plasma:

$$\Omega^2 = \prod_\alpha |\omega_\alpha|.$$

It is remarkable that one more integral of Eqs. (8)–(11) is found to exist:

$$\left[\frac{2W}{M} + \frac{1}{M}\sum_\alpha \omega_\alpha J_\alpha - \frac{1}{4}\Omega^2 l^2 - \left(\frac{dl}{dt}\right)^2 \right] l^2(t)$$
$$\equiv \sum_\alpha \mu_\alpha V_\alpha^2 l^2 = \text{const}, \quad 2W \equiv \sum_\alpha m_\alpha N_\alpha \langle v^2\rangle_\alpha, \tag{15}$$

where W is the total transverse kinetic energy of plasma per unit length of the column, and the moments V_α^2 can be treated as a spread of the velocities of particles of sort α with respect to a certain fluid velocity that is proportional to the distance from the column axis r,

$$V_\alpha^2 \equiv \langle (\mathbf{v} - \mathbf{u}_\alpha)^2 \rangle_\alpha$$
$$\mathbf{u}_\alpha(\mathbf{r},t) \equiv r\left[l^{-1}\frac{dl}{dt}\mathbf{e}_r - \frac{\omega_\alpha}{2}\mathbf{e}_\psi \right]. \tag{16}$$

Note that according to Eq. (16) the radial components of fluid velocities $\mathbf{u}_\alpha$ are equal for both sorts of particles. At the same time, the angular components that are proportional to the magnetic field strength B depend on the mass and charge of particles.

The integral (15) makes it possible to reduce the equation for the plasma spatial scale (14) to

$$l^3 \frac{d^2}{dt^2} l + \frac{1}{4}\Omega^2 l^4 = \sum_\alpha \mu_\alpha V_\alpha^2 l^2 = \text{const}, \tag{17}$$

where the constant value on the right is determined by the initial distribution functions of the particles.

Specifically, in the simplest case of stationary magnetic field the total plasma kinetic energy is found to be constant, and Eq. (17) has a stationary solution,

$$l^2 = l_{st}^2 \equiv \frac{2}{M\Omega^2}\left[2W + \sum_\alpha \omega_\alpha J_\alpha \right] = \text{const}, \tag{18}$$

that is determined by the magnetic field strength B, total transverse plasma kinetic energy W, and total canonical angular momentums J_α. If initial conditions are different from those defined by Eq. (18), i.e.,

$$l(t=0) \neq l_{st} \quad \text{and/or} \quad dl/dt \neq 0,$$

the plasma cross section oscillates harmonically in time

$$\frac{d^2}{dt^2} l^2 + \Omega^2 [l^2 - l_{st}^2] = 0. \tag{19}$$

The frequency Ω of these oscillations does not depend on their amplitude and is equal to the low-hybrid frequency for a sufficiently dense plasma.

The oscillations can also be excited from a stationary state due to temporal variation of magnetic field. However, when the strength of magnetic field varies slowly in time, so that

$$\frac{d\Omega}{dt} \ll \Omega^2,$$

a quasistationary state of the plasma column is sustained. In this case the parameters of the column follow the well-known adiabatic law

$$\Omega l_{st}^2 = \text{const}, \quad W/\Omega = \text{const}.$$

III. EXACT SOLUTION

The method of moments considered in Sec. II of the paper does not allow for a complete description of the dynamics of the plasma column in a homogeneous magnetic field. For example, it leaves unknown temporal evolution of the kinetic energy of each plasma component

$$W_\alpha \equiv \frac{1}{2} m_\alpha N_\alpha \langle v^2 \rangle_\alpha .$$

At the same time, rather complicated relationships between different moments of the distribution functions (15) result in some difficulties in setting the values of these moments at the initial moment of time. It is clear that these problems will not arise if an exact solution of the system of Vlasov kinetic equations (3) and (4) is found. Such a solution is closely connected with the integrals (13) and (15). Similarly to [22] it can be represented through arbitrary functions of two arguments:

$$f_\alpha(\mathbf{v},\mathbf{r},t)=F_\alpha(G_\alpha,g_\alpha), \quad \alpha=1,2; \tag{20}$$

$$G_\alpha=\frac{(\mathbf{v}-\mathbf{u}-\mathbf{w}_\alpha)^2}{V_\alpha^2(t)}+\frac{r^2}{l^2(t)},$$

$$g_\alpha=\frac{\mathbf{e}_z\cdot[\mathbf{r}\times(\mathbf{v}-\mathbf{w}_\alpha)]}{V_\alpha(t)l(t)}, \quad V_\alpha(t)l(t)=\text{const}, \tag{21}$$

$$\mathbf{u}(\mathbf{r},t)\equiv\frac{r}{l}\frac{dl}{dt}\mathbf{e}_r, \quad \mathbf{w}_\alpha(\mathbf{r},t)\equiv-\frac{\omega_\alpha}{2}r\mathbf{e}_\psi, \tag{22}$$

where the parameter of the solution l is nothing but the plasma spatial scale Eq. (7). Consequently, it is governed by Eq. (17). The parameters V_α are treated as spreads of the velocities of particles of sort α with respect to a certain fluid velocity, which coincides with Eq. (16)

$$V_\alpha^2\equiv\langle(\mathbf{v}-\mathbf{u}-\mathbf{w}_\alpha)^2\rangle_\alpha.$$

Note that the quasineutral approximation generally demands F_α to satisfy only one integral condition

$$\sum_\alpha Z_\alpha\int F_\alpha(\mathbf{v},\mathbf{r},t)d\mathbf{v}=0. \tag{23}$$

The solution (20)–(22) allows one to calculate the spatial distribution of the charge-separated electric field potential:

$$\sum_\alpha\frac{1}{M_\alpha}e\varphi(r,t)=-\sum_\alpha\frac{1}{Z_\alpha N_\alpha}\left[\frac{V_\alpha^2(t)}{l^2(t)}+\frac{\omega_\alpha^2}{4}\right]\frac{r^2}{2}. \tag{24}$$

The result (24) corresponds to a uniform density of spatial charge, that contradicts the starting assumption of plasma quasineutrality (4) at the periphery of the column where the plasma density is low. Therefore, the solution (20)–(22) is not quite correct everywhere in space. However, the most credible speculation is that a perturbation of this solution is considerable only at the column periphery if the plasma is dense enough. At least such a result was earlier obtained in numerical simulations of semi-infinite plasma expansion into a vacuum [3] and of an equilibrium plasma column in a homogeneous magnetic field [28]. It should be noted that the solution found in [28] is very close to the stationary case of Eqs. (20)–(22) everywhere in space with the exception of a thin layer near the column boundary.

Example

A specific example of the exact solution (20)–(22) in which the condition (23) can be easily fulfilled is the case where the distribution functions f_α have the same form and depend on one argument only. In particular, initially isotropic Maxwellian distributions of electrons and ions over velocities are considered below ($\alpha=e,i$),

$$f_\alpha(v_z,\mathbf{v},\mathbf{r},t=0)=\lambda_\alpha\exp\left(-\frac{r^2}{l_0^2}-\frac{m_\alpha v_z^2}{2T_{0\alpha}}\right)\exp\left(-\frac{m_\alpha\mathbf{v}^2}{2T_{0\alpha}}\right), \tag{25}$$

$$\lambda_\alpha\equiv\frac{N_\alpha}{\pi l_0^2}\left(\frac{m_\alpha}{2\pi T_{0\alpha}}\right)^{3/2},$$

where l_0 is the initial spatial scale of the plasma column, and $T_{0\alpha}$ is the initial temperature of particles of sort α. In this case the solution of Eqs. (3) and (4) is

$$f_\alpha(v_z,\mathbf{v},\mathbf{r},t)=\lambda_\alpha\exp\left(-\frac{r^2}{l^2(t)}-\frac{m_\alpha v_z^2}{2T_{0\alpha}}\right)\times\exp\left(-\frac{m_\alpha(\mathbf{v}-\mathbf{u}-\omega_\alpha\kappa r\mathbf{e}_\psi)^2}{2T_\alpha(t)}\right), \tag{26}$$

$$T_\alpha(t)=T_{0\alpha}\left[\frac{l_0}{l(t)}\right]^2, \quad \mathbf{u}(\mathbf{r},t)\equiv\frac{r}{l}\frac{dl}{dt}\mathbf{e}_r, \tag{27}$$

$$2\kappa\equiv\frac{B_0l_0^2}{B(t)l^2(t)}-1, \tag{28}$$

where B_0 is the initial strength of the magnetic field and $l(t)$ is governed by Eq. (17),

$$l^3\frac{d^2}{dt^2}l+\frac{1}{4}\Omega^2l^4=\frac{1}{4}\Omega_0^2l_0^4+\frac{2l_0^2}{M}\sum_\alpha N_\alpha T_{0\alpha} \tag{29}$$

under the following initial conditions:

$$l(t=0)=l_0, \quad \left.\frac{dl}{dt}\right|_{t=0}=0, \quad \Omega_0\equiv\Omega(t=0).$$

In the simplest case of a stationary magnetic field, Eq. (29) describes harmonic oscillations of the plasma column:

$$l^2=l_0^2+\frac{4W}{M\Omega^2}[1-\cos(\Omega t)], \quad W=\sum_\alpha N_\alpha T_{0\alpha}. \tag{30}$$

IV. DISCUSSION

It is expedient to discuss applicability of the solution (20) found to the description of the actual dynamics of a plasma column in a homogeneous magnetic field. As was mentioned above, the quasineutral approximation is violated at the column periphery. However, it can be used to describe plasma dynamics in the bulk of the column if the plasma is sufficiently dense there. Specifically for the above considered example with Maxwellian distribution functions for electrons and ions Eq. (26), the following inequality should be satisfied:

$$N_e\gg\frac{1}{2e^2}\left|T_e-\frac{m_e}{m_i}T_i+\frac{m_e\omega_e^2}{8}\left(\frac{B_0l_0^2}{B}+l^2\right)\right|. \tag{31}$$

On the other hand, the obtained solution corresponds to the presence of a certain distribution of angular current. It generates the intrinsic electromagnetic field of the plasma, which has not been taken into account in the basic model considered. In particular, it is possible to evaluate this field for the solution (26) since the spatial distribution of the plasma angular current in this case is expressed in a simple form:

$$\mathbf{j}(\mathbf{r},t)\equiv\sum_{\alpha} Z_\alpha e n_\alpha \omega_\alpha \kappa r \mathbf{e}_\psi . \tag{32}$$

The estimates show that the effect of the electromagnetic field caused by this current can be neglected if the following inequality holds:

$$e^2 N_e \ll m_e c^2. \tag{33}$$

It is important that the inequalities (31) and (33) can be fulfilled simultaneously.

CONCLUSION

A complete description of the dynamics of the plasma column in the homogeneous magnetic field has been given within the kinetic model. In the case of a stationary magnetic field the column cross section is found to oscillate harmonically at the low-hybrid frequency. The method applied for the analysis is very fruitful and can be used for investigation of plasma dynamics in a number of other cases.

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